EXERCISE – III HINTS & SOLUTIONS

Sol.1 A (0, 7, 10) ; B(−1, 6, 6) ; C (−4, 9, 6)

$$\left.\begin{aligned} AB &= \sqrt{1+1+16} = \sqrt{18} \\ AC &= \sqrt{16+4+16} = 9 \\ BC &= \sqrt{9+9+10} = \sqrt{18} \end{aligned}\right\} AB = BC \text{ so isoceles } \Delta.$$

Sol.2

$$G = \left(\frac{0+0+1+1}{4}, \frac{0+1+0+1}{4}, \frac{0+1+1+0}{4}\right)$$

$$= \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

Sol.3 Point equidistant from the points is center of tetrahedron.

Sol.4

α 1

(3,5,−7) P (−2,1,8)

$$P\left[\frac{-2\alpha+3}{\alpha+1}, \frac{\alpha+5}{\alpha+1}, \frac{8\alpha-7}{\alpha+1}\right]$$

$$\frac{-2\alpha+3}{\alpha+1} = 0 \Rightarrow \alpha = 3/2$$

$$P\left(0, \frac{13}{5}, 2\right)$$

Sol.5 QP = (4, −4, −2) = 2 (2, −2, −1)
So direction Ratio of line = (2, −2, −1)

$$\text{direction cosine} = \left(\frac{2}{3}, \frac{-2}{3}, \frac{-1}{3}\right)$$

Sol.6 $\ell + m + n = 0$ & $\ell^2 + m^2 = n^2$ (given)
$\ell = (m + n)$ put in IInd relatiin

(i) $m = 0$

$$\Rightarrow \ell + n = 0$$

$$\Rightarrow \frac{m}{0} = \frac{\ell}{1} = -\frac{n}{1}$$

$$= \frac{\sqrt{\ell^2+m^2+n^2}}{\sqrt{O^2+1^2+\ell^2}} = \frac{1}{\sqrt{2}}$$

(ii) $m + n = 0$

$$\Rightarrow \frac{m}{1} = \frac{n}{-1} = \frac{\ell}{0}$$

$$\Rightarrow \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{\ell^2+1^2+0}} = \frac{1}{\sqrt{2}}$$

$$\text{So } (\ell_1, m_1, n_1) = \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right);$$

$$(\ell_1, m_2, n_2) = \left(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right);\ \theta = 60^\circ$$

Sol.7 Equation of line joining A & B

$$\frac{x+9}{-20} = \frac{y-4}{4} = \frac{z-5}{6} = \ell \text{ (Let)}$$

Let a point C oin the line is
$(-20\lambda - 9, 4\lambda + 4, 6\lambda + 5)$
Now Co (where O is origin)

$\overrightarrow{CO} = (-20\lambda - 9, 4\lambda + 4, 6\lambda + 5)$

& $\overrightarrow{CO} \cdot \overrightarrow{AB} = 0$ ($\because$ $\overrightarrow{CO}$ is $\perp$r to line)

$\Rightarrow 400\lambda + 180 + 16\lambda + 16 + 36\lambda + 30 = 0$

$$\lambda = -\frac{1}{2}$$

So point C = (1, 2, 2)
which is also the mid point of A & B.

Sol.8 $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$;
where $\beta = 120^\circ$, $\gamma = 135^\circ$

$$\text{So } (\ell, m, n) = \left(\frac{1}{2}, -\frac{1}{2}, \frac{-\sqrt{3}}{2}\right), = \vec{d} \text{ (say)}$$

$$\text{Projected } \overrightarrow{PQ} \text{ on } \vec{d} = \frac{\overrightarrow{PQ} \cdot \vec{d}}{|\vec{d}|} = 2 - 2\sqrt{3}$$

Sol.9 Let the equation of plane is
Ax + By + Cz + 1 = 0
using points (1, 0, 0) & (0, 1, 0)
A = −1 & B = −1

$$\text{\& angle ; } \frac{1}{\sqrt{2}} = \frac{A.(1) + B.(1)}{\sqrt{1^2+1^2+C^2}.\sqrt{2}}$$

$$\Rightarrow C = \pm\sqrt{2}$$

Sol.10 Let $\vec{a}$ (1,1,1); $\vec{b}$ (1,–1,1)& $\vec{c}$ (–7, –3, –5)

normal of the plane

$\vec{n}_1 = (\vec{b} - \vec{a}) \times (\vec{b} - \vec{c})$

& $\vec{n}_2 = (0, 1, 0)$

angle = $\frac{\pi}{2}$

Sol.11 Equation of L_1 : $\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$

& of L_2 : $\frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$ ($\because$ || lines)

Equation of any plane through L_1

a(x – 4) + b (y – 3) + C (z – 2) = 0 ...(i)

where a – 4b + 5c = 0 ...(ii)

also (3, –2, 0) lie oin plane (i)

using (ii) & (iii)

a + 5b + 2c = 0 ...(iii)

$\frac{a}{11} = \frac{b}{-1} = \frac{c}{-3}$

So Equation of plane 11x – y – 3z = 35

Sol.12 Line and plane are parallel.

So image of (1, 2, –3) about the plane

3x – 3y + 10z = 26

is (4, –1/7)

So equation of line is

$\frac{x-4}{9} = \frac{y+1}{-1} = \frac{z-7}{-3}$

Sol.13 $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \alpha$

p $(2\alpha + 1, 3\alpha + 2, 4\alpha + 3)$

$2\ \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \mu$

$(5\mu + 4, 2\mu + 1, \mu)$

$2\alpha + 1 = 5\mu = 4$

$3\alpha + 2 = 2\mu = 1$

$4\alpha + 3 = \mu$

a = –1

P(–1, –1, –1)

Similarly

PoI of other two lines

Q(4, 0, –1)

PQ = $\sqrt{26}$

Sol.14 Certre $\left(0, \frac{1}{2}, 1\right)$

Diameter= $\sqrt{(2+2)^2 + (1-1-2)^2 + (4+2)^2} = \sqrt{61}$

$\Rightarrow \alpha = \frac{\sqrt{61}}{2}$

Eq^n of sphere

$(x - 0)^2 + \left(y - \frac{1}{2}\right)^2 + (z - 1)^2 = \frac{61}{4}$

$\Rightarrow x^2 + y^2 + z^2 - y - 2z - 14 = 0$

Sol.15 π_1 : 2x + 3y – z + 1 = 0 ; $\vec{n}_1$ = (2, 3, –1)

π_2 : x + y – 2z + 3 = 0 ; $\vec{n}_2$ = (1, 1, –2)

Let the equation of the required polane :

$\pi = \pi_1 + \lambda\pi_2$ ...(i)

& normal of 5 plae is $(2 + \lambda, 3 + \lambda, -1 - 2\lambda) = \vec{n}$

also fnπ_3 : 3x – y – 2z = 4 ; $\vec{n}_3$ = (3, –1, –2)

& $\vec{n} \cdot \vec{n}_3 = 0 \Rightarrow \lambda = -\frac{5}{6}$

Put in (i) plane is 7x + 13y + 4z – 9 = 0

Sol.16 Line of intersection of planes

2x + y = 0 & x – y + z = 0

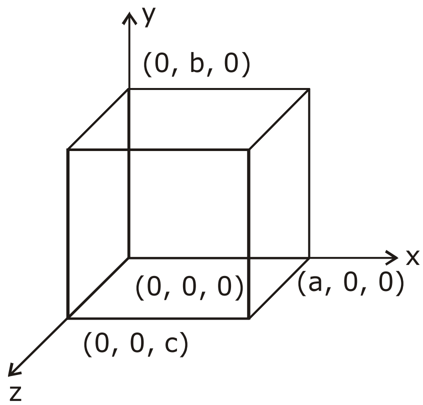
Sol.17 Point of intersection of line & plane

= (2, –1, 2)

using distance formula

$\alpha = -1, \frac{80}{63}$

Sol.18 Angle between diagonals



$$= \cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}\right)$$

Sol.19 $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

lines will be coplanar so

$$\begin{vmatrix} a & b & c \\ 2 & 1 & 1 \\ 3 & 3 & 0 \end{vmatrix} = 0 \Rightarrow a + b + c$$

$$\cos 60° = \left|\frac{2a+b+c}{a^2+b^2+c^2\sqrt{6}}\right|$$

$2b^2 + 2c^2 + 5bc = 0$

$(b + 2c)(2b + c) = 0$

$b = -2c$ or $b = -c/2$

$a = -c$ or $a = c/2$

$$\frac{x}{-c} = \frac{y}{-2c} = \frac{z}{c} \text{ or } \frac{x}{c/2} = \frac{y}{-c/2} = \frac{z}{c}$$

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{-1} \quad \text{or} \quad \frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$$

Sol.20 Equation of plane through given line's

$(3x - y + 2z - 1) + \lambda(x + 2y - z - 2) = 0$ ...(i)

This is perpoendicular to

$3x + 2y + z = 0$ ...(ii)

So $\lambda = -\frac{2}{3}$

Putting this in (i) ;

$3x - 8y + 7z + 4 = 0$ ...(iii)

using (ii) & (iii) equation of line.

Sol.21 $\cos\theta = \left|\frac{\ell m + mn + n\ell}{\ell^2 + m^2 + n^2}\right|$

using the given equation

$$\theta = \cos^{-1}\left(\frac{4}{9}\right)$$

Sol.22 M(1, 0, 5) & N (–2, 0, 4)

Equation of MN

$$\frac{x-1}{3} = \frac{y-0}{0} = \frac{z-5}{1}$$

angle between line & plane is

$$\sin\theta = \frac{(3, 0, 1).(1, 1, 1)}{\sqrt{3^2+1^2}.\sqrt{1^2+1^2+1^2}}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{4}{\sqrt{30}}\right)$$

Sol.23 $x = 0;\ \frac{y}{b} + \frac{z}{c} = 1$

is a line in (y – z) plane with y intercept 'b' & zintercept 'c'.

$y = 0;\ \frac{x}{a} - \frac{z}{c} = 1$

is a line in (x – z) plane with x intercept 'a' & z intercept '–c'.

So using distancer between two skew lines

$$\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}.$$

Sol.24 Now plane passing through origin

Normal of plane = $\vec{a} \times \vec{b}$

= (1, 2, –3) × (2, –3, 1)

$= -7\hat{i} - 7\hat{j} - 7\hat{k}$

$= -7(\hat{i} + \hat{j} + \hat{k})$

so Eq^n of plane is

$x + y + z = 0$